

An Investigation into Road Capacity

In some consistent set of units let

v = velocity car

d_v = recommended stopping distance

between cars at speed v

g = length of car

c_v = capacity road at velocity v

Now it is easily demonstrated that

$d_v = av^2 + bv$ where a and b are constants

in some defined and consistent set of units.

To determine the capacity of the road we

thus assume a distance between cars of d_v

plus the length of the car g .

Now consider the time it takes for one car

to pass a fixed point at velocity v

$t_v = (av^2 + bv + g) / v$ and hence

$c_v = v / (av^2 + bv + g)$

To determine the maximum value of this

function we equate dc_v / dv to zero.

$dc_v / dv =$

$\{(av^2 + bv + g) - v(2av + b)\} / (av^2 + bv + g)^2$

Hence $(av^2 + bv + g) = v(2av + b)$

and rearranging we quickly deduce

$v_{\max} = \sqrt{(g/a)}$

which is a pretty neat result.

As an aside the term $(av^2 + bv + g)^2$ must

always be positive so we avoid spurious

results where $dc_v / dv \rightarrow \infty$

Values of constants

Highway Code stopping distances given as

mph	20	30	40	50	60
Thinking	20	30	40	50	60
Stopping	20	45	80	125	180

where distances are in feet .

Hence $a = 0.05$ and $b = 1$

though interestingly we don't need this value

to determine $v_{\max}$

Assume $g = 15$ feet

$v_{\max} = \sqrt{(g/a)}$

$\sqrt{(15/0.05)}$

17.3 mph.

As a check if we use a graphical calculator to

identify the maximum value of the function

$v / (0.05v^2 + v + 15)$ and use the

conversion

factor 88 feet / sec = 60 mph we obtain

again $v_{\max} = 17.3$ mph again

Maximum Capacity

Inserting $\sqrt{(g/a)}$ into the function for c_v

we obtain $c_{v\max} = \sqrt{(g/a)} / \{2g + b\sqrt{(g/a)}\}$

Inserting actual values and multiplying up to

obtain maximum capacity / hour

$c_{v\max} = 60 \times 60 \times 17.3 / (2 \times 15 + 17.3)$

≈ 1300 / hour

which seems a not unreasonable result.

Summary

As motorways start to "clog" the tactic is to reduce speed to increase capacity.

However once a "local" speed drops below a critical value $v_{\max}$ queues will build up for

no external valid reason. You join a queue,

crawl for some period and then peel off the

front. These queues move backwards slowly

down the motorway . ∞ rg

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