Trapezoidal Numbers

The sum of any consecutive sequence is called a trapezoidal number (why?).

eg 8 + 9 + 10 + 11 = 38

This can be considered as the difference of two triangular numbers. $T_{11} - T_7$ *(why?)*

Let the two triangular numbers be c and d

$T_c - T_d =$

$\frac{1}{2} c (c + 1) - \frac{1}{2} d (d + 1)$

$= \frac{1}{2} (c^2 - d^2) + \frac{1}{2} (c - d)$

Which probably doesn't get us anywhere.

But tackle the problem more directly.

Take a sequence from a to b

$a + (a + 1) + (a + 2) + (a + 3) \ldots.$

$b - 2 + b - 1 + b$

repeat in reverse

$b + (b - 1) + (b - 2) + (b - 3) \ldots.$

$(a - 2) + (a - 1) + a$

which added together gives

$(b - a + 1)$ lots of $(a + b)$

Now complete this table

a	b	(b – a +1)	(a + b)
even	even	odd	even
even	odd	even	odd
odd	even	even	odd
odd	odd	odd	even

So all trapezoidal numbers must have an odd factor so cannot be of the form 2^n ie sum to 2, 4, 8, 16 etc.

Addendum

Years later I realised there is a much quicker way.

For sequences of odd length 2n+1 the middle number is an integer m so the sum is m (2n+1) which must contain an odd factor.

For sequences of even length 2n the "middle" number is of the form m + ½ so the sum is

$2n (m + \frac{1}{2}) = n (2m +1)$

which also must contain an odd factor. Easy! ∞ *rg*

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Addendum

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For sequences of odd length $2n+1$ the middle number is an integer m so the sum is $m\,(2n+1)$ which must contain an odd factor.

For sequences of even length $2n$ the "middle" number is of the form $m + \tfrac{1}{2}$ so the sum is

$2n\,(m + \tfrac{1}{2}) = n\,(2m + 1)$

which also must contain an odd factor. Easy! *rg*