Mr G’s Little Book on

Converting between Fahrenheit and Celsius

Preliminaries

The equations we will be using are

The standard equation for a straight line

$y = mx + c$

and

Given a point and a gradient the equation is

$y - y_1 = m (x - x_1)$

The Fahrenheit Scale

To determine a temperature scale Fahrenheit needed to fixed points which he labelled 0 and 90. He chose the coldest brine he could create – water plus salt plus ice and his own body temperature.

His calculations were incorrect but the principles remained and the scale underwent several redefinitions.

Details of Conversion

Suppose some line exists that converts °C to °F. It will be of the form

$F = mC + k$

To determine m and k we need two pairs of values.

We know 32°F = 0°C so

$32 = m0 + k$

which immediately gives $k = 32$.

Also 100°C = 212°F so

$212 = 100m + 32$ so

$100m = 180$ giving

$m = 9/5$.

so $F = 9/5C + 32$

This can be interpreted as a spread of 5°C is equivalent to 9°F – that is the Fahrenheit scale has greater discrimination.

Now assume there is some temperature t that has the same measure in both F and C.

So $t = 9/5\ t + 32$

$t - 9/5\ t = 32$

$- 4/5\ t = 32$

therefore $t = - 5\times32 / 4$

$t = - 40$ that is

$- 40°C = - 40°F$

Now if we wanted to represent the Celsius scale and the Fahrenheit scale by two straight lines that intersected at – 40, what would those lines look like?

The intersection is (– 40, – 40) so we need two gradients to determine.

So let $m_C = 5/5$ and $m_F = 9/5$

So for the Celsius line the equation is

$(y_C - {}^-40) = m_C\ (x_C - {}^-40)$

$y_C + 40 = m_C (x_C + 40)$

but we have set $m_C = 1$ so

$$y_C = x_C$$

A bit disappointing but inevitable.

We just have a straight line through the origin at 45°.

(Note here we are using the degree term in two completely different contexts).

Now for the Fahrenheit line.

$(y_F - {}^{-}40) = m_F (x_F - {}^{-}40)$

$y_F + 40 = m_F (x_F + 40)$

and we have set $m_F = 9/5$ so

$$y_F + 40 = 9/5 (x_F + 40)$$

$$y_F + 40 = 9/5\, x_F + 72$$

$$y_F = 9/5\, x_F + 32$$

which is the same equation we derived before. It has the proportionate gradient to C and intersects the y-axis at 32.

Another Approach

Suppose we approach directly and just asked "Is there some constant A that we can add to °C, multiply by 9/5 and then subtract the same constant to give the same answer?

So $C = (F - 32) \times 5/9$ which is the traditional approach.

$(F - 32) \times 5/9 = [\ (F + A) \times 5/9\ ] - A$

$5/9\,F - 160/9 = 5/9F + 5/9A) - A$

cancelling out the 5/9F gives

$- 160/9 = 5/9A - A$

$- 160/9 = - 4/9A$

and we determine $A = 40$.

That is adding 40 multiplying and then subtracting 40 gives the same result.

Summary

Given we have two lines intersecting at $(-40 , -40)$ our first action is to move that coordinate system to the intersection of the two lines

ie we add 40 to the given value.

We now have a line of direct proportion

$F' \propto C'$

$F' = k\,C'$

where $k = 9/5$

We perform the calculation and then move the axis back to their original position.

Easy!

rg

My acknowledgment to a great Physics Teacher Holly Berry at Spalding Grammar School who taught me this method.

Addendum - Another Way

I then realised I could analyse this using vectors.

Let L (A,A) be the coordinates to be found.

M (0,32) and N (100,212)

gradient Vector MN is 180/100 = 9/5.

Vectors LM and MN are collinear.

Therefore gradient LM = 9/5.

So $(32 - A) / (0 - A) = 9/5$

So $5 (32 - A) = -9A$

giving $A = -40$