Mr. G's Handbook of Mathematics

Venn Diagrams

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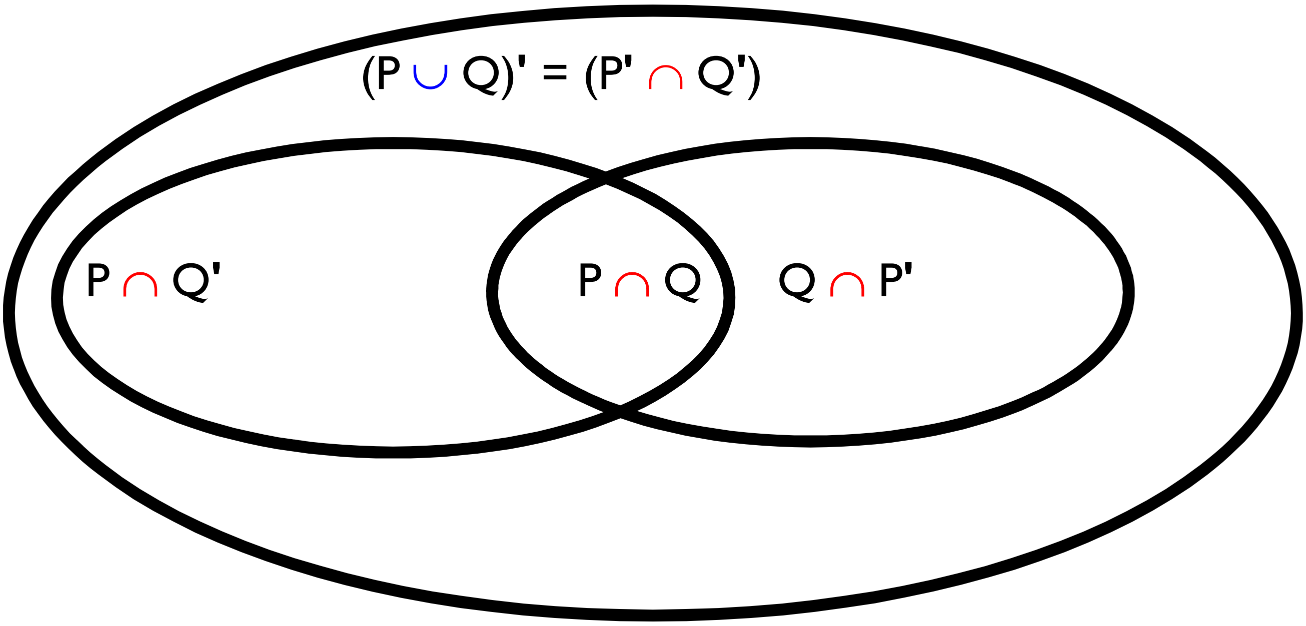
Venn Diagrams 1

$P \cap Q$ say P **intersection** Q means P **AND** Q

$P \cup Q$ say P **union** Q means P **OR** Q **or both**

$$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$$



where the left ellipse is (P) and the right ellipse is (Q)

Thus $n(P) = n(P \cap Q') + n(P \cap Q)$

and $n(Q) = n(Q \cap P') + n(Q \cap P)$

hence $n(P \cap Q) = n(P) - n(P \cap Q')$

$n(P \cap Q) = n(Q) - n(P' \cap Q)$

From diagram $(P \cup Q)' = (P' \cap Q')$ †

$(P \cap Q)' = (P' \cup Q')$ †

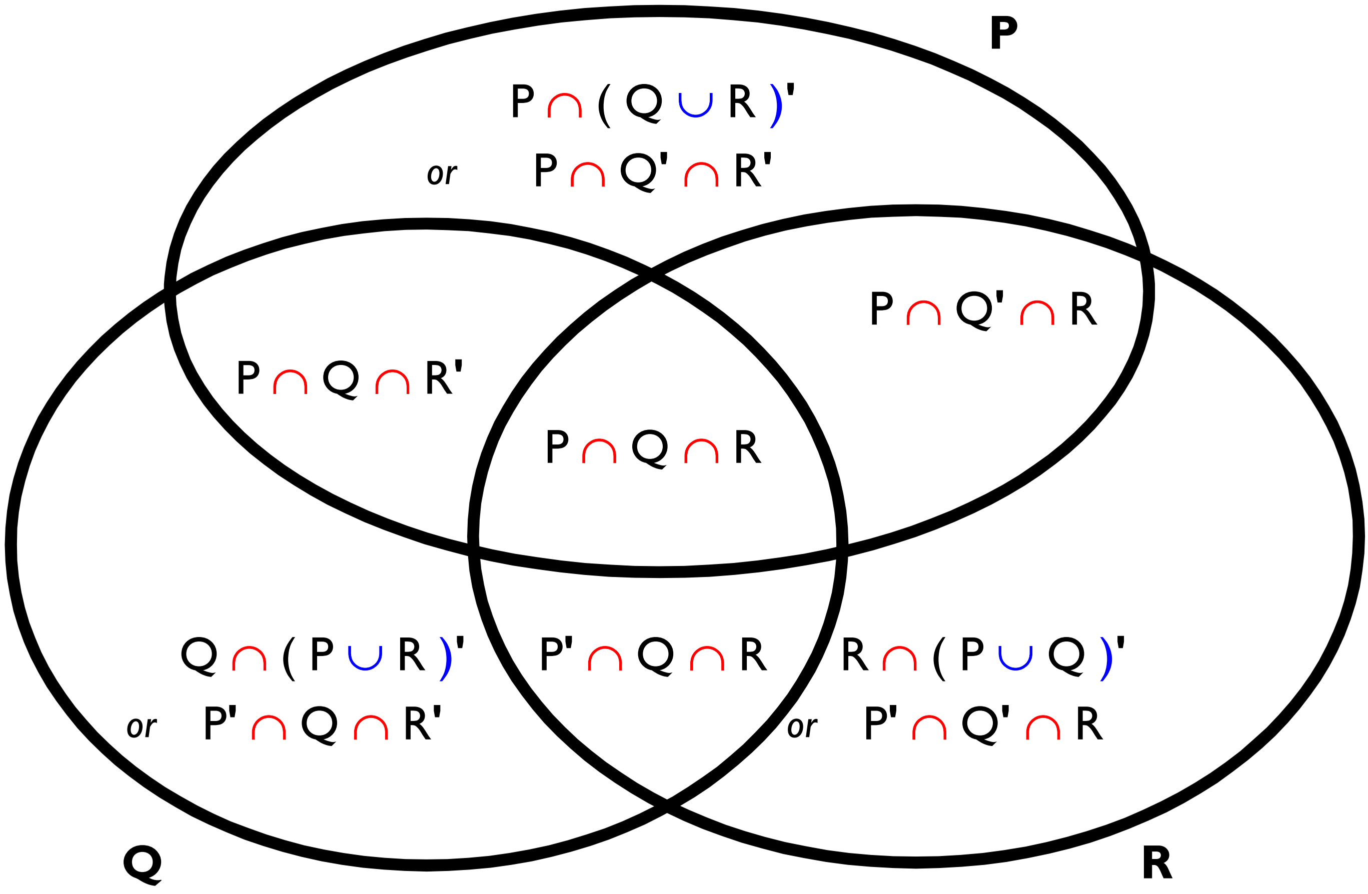
Notes

There is also the term P \ Q (say "P diff Q") $\equiv P \cap Q'$

It immediately follows that $P \setminus Q \equiv Q' \setminus P'$ *which is similar to the contrapositive rule.*

† *these are de Morgan's Rules*

Venn Diagrams 2



$n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(P \cap Q) - n(P \cap Q) + n(P \cap Q \cap R)$

Other Set Relationships

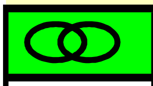
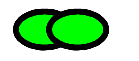
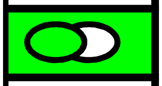
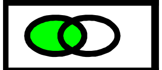
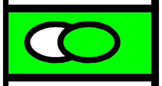
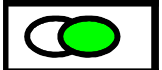
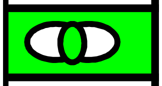
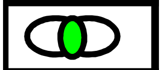
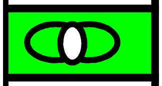
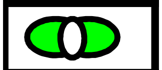
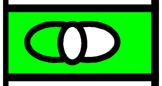
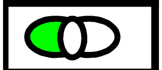
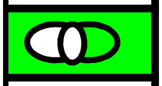
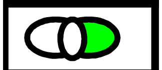
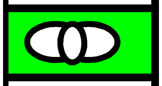
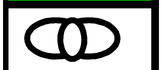
if $(A \cap B) = B$ then $B \subseteq A$

or $(A \cup B) = A$ then $B \subseteq A$

if $(A \cap B) = A$ then $A \subseteq B$

or $(A \cup B) = B$ then $A \subseteq B$

Notes

Venn Diagrams			Truth Tables		Logic Gates	
∪ union ∩ intersection		*de Morgan*	T T F F T F T F	**p** **q**	∨ **OR** ∧ **AND**	$\underline{\vee}$ **XOR** $\underline{\wedge}$ **XAND**
	U	ε	T T T T	*truth*	p ∨ ¬p	¬(p ∧ ¬p)
	(P ∪ Q)	(P' ∩ Q')'	T T T F	*incl. disjunction* †	p ∨ q	¬(¬p∧¬q)
	(P ∪ Q')	(P' ∩ Q)'	T T F T	*implication*	p ∨ ¬q	¬(¬p ∧ q)
	P		T T F F	*p*	p ∨ p	¬(¬p∧¬p)
	(P' ∪ Q)	(P ∩ Q')'	T F T T	*implication*	¬p ∨ q	¬(p ∧ ¬q)
	Q		T F T F	*q*	q ∨ q	¬(¬p∧¬q)
	(P ∩ Q) ∪ (P' ∩ Q')		T F F T	*equivalence/ IFF*	¬(p $\underline{\vee}$ q)	p $\underline{\wedge}$ q
	(P' ∪ Q')'	P ∩ Q	T F F F	*conjunction*	¬(¬p∨¬q)	p ∧ q
	(P' ∪ Q')	(P ∩ Q)'	F T T T	*incompatible*	¬p ∨ ¬q	¬(p ∧ q)
	(P ∩ Q') ∪ (P' ∩ Q)		F T T F	*excl. disjunction*	p $\underline{\vee}$ q	¬(p $\underline{\wedge}$ q)
	Q'		F T F T	*negation*	¬(q ∨ q)	(¬q ∧ ¬q)
	(P' ∪ Q)'	P ∩ Q'	F T F F	*non implication*	¬(¬p ∨ q)	p ∧ ¬q
	P'		F F T T	*negation*	¬(p ∨ p)	¬p ∧ ¬p
	(P ∪ Q')'	P' ∩ Q	F F T F	*non implication*	¬(p ∨ ¬q)	¬p ∧ q
	(P ∪ Q)'	P' ∩ Q'	F F F T	*joint denial*	¬(p ∨ q)	¬p ∧ ¬q
	∅		F F F F	*contradiction*	¬(p ∨ ¬p)	(¬p ∧ p)

Notes

† *also termed "exhaustiveness"*

¬(p ∨ q) *is the same as* (¬p ∧ ¬q) *(everything gets multiplied by "¬")*

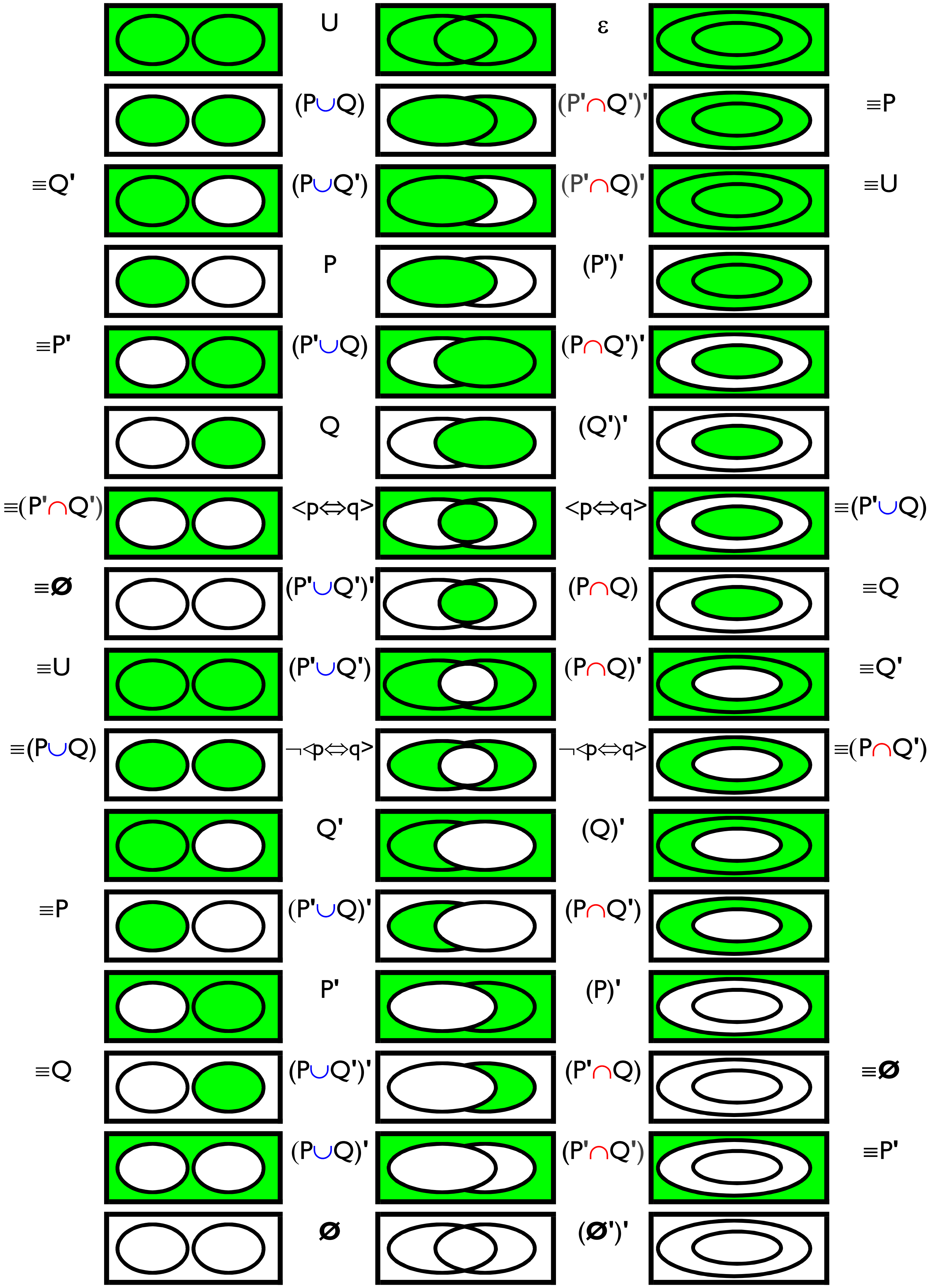
If you look carefully, the colouring follows the Truth table exactly.

1 st column colours the intersection — *2 nd column coulours P*

3 rd column colours Q — *4 th column colours the outside*

so P ∩ Q , P ∩ Q', Q, P' ∩ Q, P' ∩ Q' *are coloured for T in each column in turn.*

Venn Diagrams for disjunction, intersection and inclusion



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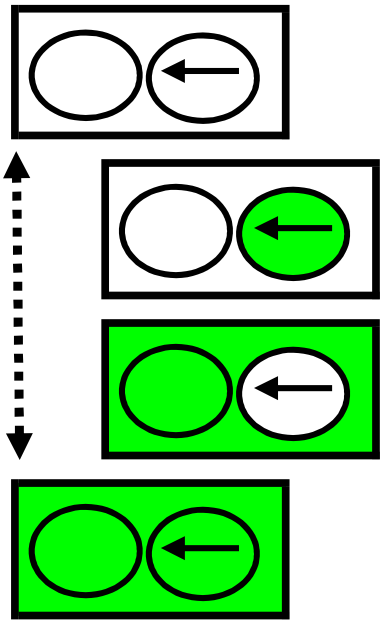
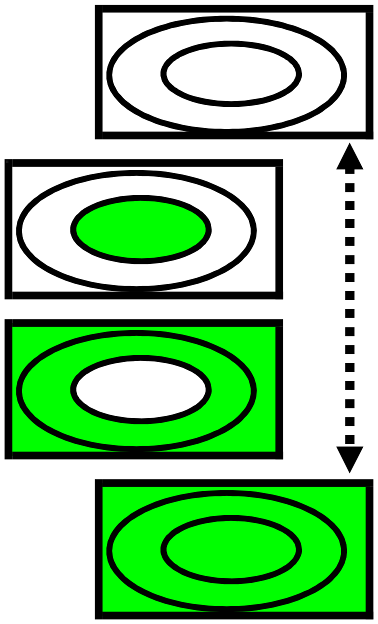
Visual Representation of de Goodhand's Rule

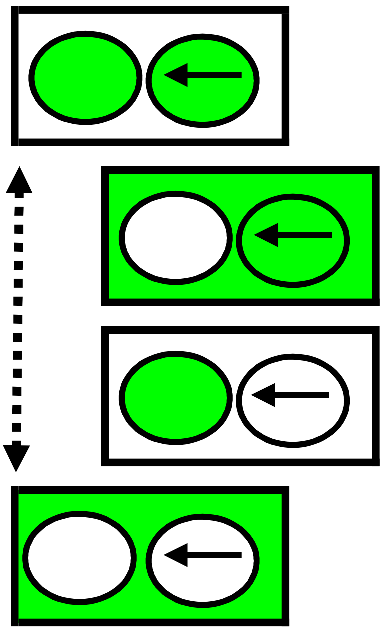
To move between disjoint and inclusive sets, change P to P ' (or P ' to P)

for the set relationships that cannot be defined by P (or P ') alone.

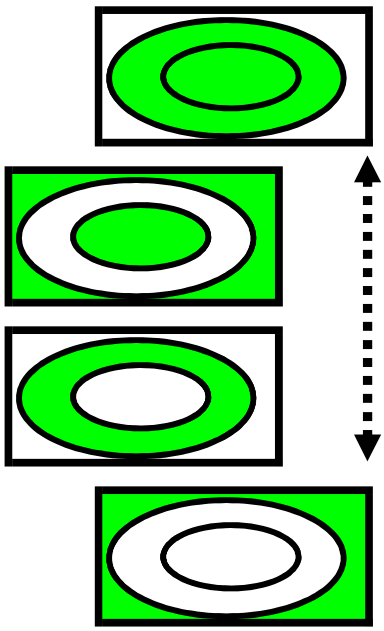


$P \cap Q$ $\varnothing$	$P' \cap Q$ $\varnothing$	7
$P' \cap Q$ Q	$P \cap Q$ Q	13
$P \cup Q'$ Q'	$P' \cup Q'$ Q'	2
$P' \cup Q'$ U	$P \cup Q'$ ε	8





$P \cup Q \longrightarrow P \cup Q$	P	1
$P' \cup Q \longrightarrow P' \cup Q$	P'	4
$P \cap Q' \longrightarrow P \cap Q'$	P	11
$P' \cap Q' \longrightarrow P' \cap Q'$	P'	14



Notes

de Goodhand's rule holds for the top two pairs with their complements via de Morgan's rules

The bottom two pairs with complements are more self evident and pair without change.

Drawing Venn diagrams for disjoint and inclusion reduces the arrangements to 2^3.

There are likewise 8 ways of writing an expression with three elements.

So there is a one-to-one a pairing between the two and some pattern must then be apparent.

Counting

No.	***Greek***	***Latin***
1	*mono*	*uni*
2	*duo*	*bi*
3	*tri*	*tri*
4	*tetra*	*quad*
5	*penta*	*quin*
6	*hexa*	*sex*
7	*hepta*	*sept*
8	*octo*	*oct*
9	*nona*	*non*
10	*deca*	*dec*

The "Handbook of Mathematics" has been written and produced by Robert Goodhand

Although the formulae and expressions given have been individually derived and checked errors do creep in. Sections of the Handbook are continuously updated.

If you would like the latest issue, just email me at robert.goodhand@gmail.com

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