Complex Tangents

Introduction

On a separate handwritten sheets I derived the following expression

tan (x + iy)

$$= \sin 2x + i \sinh 2y \;/\; \cos 2x + \cosh 2y$$

tanh (x + iy)

$$= \sinh 2x + i \sin 2y \;/\; \cosh 2x + \cosh 2y$$

which have a beautiful symmetry about them.

However after ploughing through all that, I realised I could have done it in two lines

Formula for tan (A + B)

Here we "cheat" slightly – knowing where we need to end up we do it in reverse

$$\sin 2A + \sin 2B \;/\; \cos 2A + \cos 2B =$$

$$2 \sin (A + B) \cos (A - B) \;/\; 2 \cos (A + B) \cos (A - B)$$

$$= \tan (A + B)$$

which is incredibly neat and doesn't show up on an internet search. So

tan (x + iy) =

$$\sin 2x + \sin 2\,iy \;/\; \cos 2x + \cos 2\,iy$$

$$= \sin 2x + i \sin 2y \;/\; \cos 2x + \cosh i\, y$$

which otherwise took me a whole page.

Formula for tanh (A + B)

The formula for circular functions translates identically for hyperbolic functions

$$\sinh 2A + \sinh 2B \;/\; \cosh 2A + \cosh 2B$$

$$= \tanh (A + B)$$

So tanh (x + iy)

$$= \sinh 2x + \sinh 2iy \;/\; \cosh 2x + \cosh 2iy$$

$$= \sinh 2x + I \sinh 2y \;/\; \cosh 2x + \cos 2y$$

Hardly worth the post.

rg

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$$\frac{2 \sin (A + B) \cos (A - B)}{2 \cos (A + B) \cos (A - B)}$$

$$= \tan (A + B)$$

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tan (x + iy) =

$$\frac{\sin 2x + \sin 2 iy}{\cos 2x + \cos 2 iy}$$

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Formula for tanh (A + B)

The formula for circular functions translates identically for hyperbolic functions

$$\frac{\sinh 2A + \sinh 2B}{\cosh 2A + \cosh 2B}$$

$$= \tanh (A + B)$$

So tanh (x + iy)

$$= \frac{\sinh 2x + \sinh 2iy}{\cosh 2x + \cosh 2iy}$$

$$= \frac{\sinh 2x + I \sinh 2y}{\cosh 2x + \cos 2y}$$

Hardly worth the post.

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