Why Pianos are always Out of Tune

If we first move up through the kepboard in 5^{ths} we get

$0^\#$	$1^\#$	$2^\#$	$3^\#$	$4^\#$	$5^\#$	$6^\#$	$7^\#$	$8^\#$	$9^\#$	$10^\#$	$11^\#$	$12^\#$
C	G	D	A	E	B	$F^\#$	$C^\#$	$G^\#$	$D^\#$	$A^\#$	F	C

and the whole stacking is seven octaves – count them.

Now freq(G) / freq(C) = $^3/_2$ so 12 intervals = $(^3/_2)^{12}$ = 129,75

and freq(C^7) / freq (C^0) = 128 or 1.365% over.

If we now move up through the kepboard in 4^{ths} we get

0^b	1^b	2^b	3^b	4^b	5^b	6^b	7^b	8^b	9^b	10^b	11^b	12^b
C	F	B^b	E^b	A^b	D^b	G^b	B	E	A	D	G	C

and the whole stacking is five octaves – count them

Now freq(F) / freq(C) = $^4/_3$ so 12 intervals = $(^4/_3)^{12}$ = 31.56

and freq(C^5) / freq (C^0) = 32 and 1.345% short.

Now because the perfect fifth – or its inversion the perfect fourth – contains the factor 3 then from the outset it's destined never to line up exactly with any multiple of octaves.

So pianos are now tuned to a tempered scale which ensures the stacking of fourths or fifths exactly fits whole octaves with the necessary adjustment taking place with the interval itself.

rg

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C	G	D	A	E	B	$F^{\#}$	$C^{\#}$	$G^{\#}$	$D^{\#}$	$A^{\#}$	F	C

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