An Investigation into Power Sums

Take any number and add together each digit raised to the power n and repeat. What happens?

After completing the investigations and armed with "solutions" of certain patterns I searched the internet to discover there are numbers called perfect digital invariants (PDI's) – number whose digits raised to a specific power sum to themselves. We always discount 1 and variants of. Numbers that cycle round are called recurring digital invariants (RDI).

Squares

The only numbers that sum to themselves are 1 10 100 etc.

Between 1 and 100 there are 20 numbers that eventually sum to 1

1 7 10 13 19 23 28 31 32 44 49 68 70 79 82 86 91 94 97 100

eg $23 \Rightarrow 2^2+3^2=1 \Rightarrow 1^2+3^2=10 \Rightarrow 1^2=1$ end

All other numbers between 1 and 100 end up in the cycle 4

$\rightarrow 16 \rightarrow 37 \rightarrow 58 \rightarrow 89 \rightarrow 145 \rightarrow 42 \rightarrow 20 \rightarrow 4$

Cubes

There are 4 PDIs - 153, 371, 370, 407

$153 = 1^3 + 5^3 + 3^3 = 153$

$371 \Rightarrow 3^3 + 7^3 + 1^3 = 371$

$370 = 3^3 + 7^3 + 0^3 = 370$

$407 \Rightarrow 4^3 + 0^3 + 7^3 = 407$

Between 1 and 100 it's 33, 28, 13 and 5 that end up at one of these 4.

There are two 2-stage RDIs -

$352 \rightarrow 160 \rightarrow 352$ and

$1459 \rightarrow 919 \rightarrow 1459$.

A third one is $136 \rightarrow 244 \rightarrow 136$ which I missed as nothing smaller cycles in to this.

I found one 3-stage RDI $133 \rightarrow 55 \rightarrow 250$ which 11 numbers under 100 feed into.

These are all the cubic PDIs and RDIs that exist.

Quartics

There are 2 PDIs – 1634 and 8208.

4 numbers that cycle into 8208 are 12, 17, 21 and 71.

I found two RD namely

$6514 \rightarrow 2178 \rightarrow 6514$ and

$13139 \rightarrow 6725 \rightarrow 4338 \rightarrow 4514 \rightarrow 1138 \rightarrow 4179 \rightarrow 9219 \rightarrow 13139$

I believe this exhausts all possibilities.

Robert Goodhand

11th November 2019

An Investigation into Power Sums

Take any number and add together each digit raised to the power n and repeat. What happens?

After completing the investigations and armed with "solutions" of certain patterns I searched the internet to discover there are numbers called perfect digital invariants (PDI's) – number whose digits raised to a specific power sum to themselves. We always discount 1 and variants of. Numbers that cycle round are called recurring digital invariants (RDI).

Squares

The only numbers that sum to themselves are 1 10 100 etc.

Between 1 and 100 there are 20 numbers that eventually sum to 1

1 7 10 13 19 23 28 31 32 44 49 68 70 79 82 86 91 94 97 100

eg $23 \Rightarrow 2^2+3^2=1 \Rightarrow 1^2+3^2=10 \Rightarrow 1^2=1$ end

All other numbers between 1 and 100 end up in the cycle 4

$\rightarrow 16 \rightarrow 37 \rightarrow 58 \rightarrow 89 \rightarrow 145 \rightarrow 42 \rightarrow 20 \rightarrow 4$

Cubes

There are 4 PDIs - 153, 371, 370, 407

$153 = 1^3 + 5^3 + 3^3 = 153$

$371 \Rightarrow 3^3 + 7^3 + 1^3 = 371$

$370 = 3^3 + 7^3 + 0^3 = 370$

$407 \Rightarrow 4^3 + 0^3 + 7^3 = 407$

Between 1 and 100 it's 33, 28, 13 and 5 that end up at one of these 4.

There are two 2-stage RDIs -

$352 \rightarrow 160 \rightarrow 352$ and

$1459 \rightarrow 919 \rightarrow 1459$.

A third one is $136 \rightarrow 244 \rightarrow 136$ which I missed as nothing smaller cycles in to this.

I found one 3-stage RDI $133 \rightarrow 55 \rightarrow 250$ which 11 numbers under 100 feed into.

These are all the cubic PDIs and RDIs that exist.

Quartics

There are 2 PDIs – 1634 and 8208.

4 numbers that cycle into 8208 are 12, 17, 21 and 71.

I found two RD namely

$6514 \rightarrow 2178 \rightarrow 6514$ and

$13139 \rightarrow 6725 \rightarrow 4338 \rightarrow 4514 \rightarrow 1138 \rightarrow 4179 \rightarrow 9219 \rightarrow 13139$

I believe this exhausts all possibilities.

Robert Goodhand

11th November 2019