12 Rules of Logarithms

Introduction

Supppose $a = b^n$

Taking logs both sides despite not yet knowing what a log is gives

$$\log_b a = \log b^n$$

Using the assumed power rule gives

$$\log_b a = n \log_b b$$

and if $\log_b b = 1$ we have $\log_b a = n$

and we can now actually define logs even though we assumed to prove!

the antilog is defined $b^n = \text{antilog}_b n$

eg $\log_{10} 100 = 2$ and $\text{antilog}_{10} 2 = 100$

antilogarithm $\equiv$ exponentiation

Inverse (exponentiation of log)

In base b $b^{\log a} = a$

Inverse (log exponentiation)

$$\log_b b^x = x$$

If not specified $\log \equiv \log_{10}$ $\ln \equiv \log_e$

Putting bases in red is just for emphasis

12 Rules of Logarithms

Identity Rule $\log_b b = 1$

Zero Rule $\log_b 0 = 1$

Product Rule

$\log_b xy = \log_b x + \log_b y$

To multiply add logs and take antilog

Quotient Rule

$\log_b x/y = \log_b x - \log_b y$

To divide subtract and take antilog

Power Rule

$\log_b a^k = k \log_b a$

and we use this relationship to solve exponential equations.

Inverse Rule

Given $\log_b 1 = 0$

$\log_b 1/y = -\log_b y$

We can find logs of fractions but NOT negative numbers unless we move to the complex domain

Log Product Rule

let $b^n = a$ and take logs both sides

$\log_c b^n = \log_c a$

However $n = \log_b a$ so substituting and then applying the power rule

$\log_b a \times \log_c b = \log_c a$

and rearrange to a logical sequence

$\log_c b \times \log_b a = \log_c a$

Base Change Rule

$\log_b a = \log_c a / \log_c b$

eg $\log_4 64 = \ln 64 / \ln 4 = 3$

Reciprocal Rule

$\log_b a = (\log_a b)^{-1}$

because from the base change rule

$\log_a b$ is the reciprocal of $\log_b a$

Power Base Rule

$$\begin{aligned} \log_{b^a} c &= \log_b c / \log_b b^a \\ &= \log_b c / a \log_b b \\ &= (\log_b c) / a \end{aligned}$$

eg $\log_4 64 = 3$ but $\log_{16} 64 = 1.5$

because $16 = 4^2$

Power Base Inverse Rule

By setting $a = -1$

$\log_{1/b} c \quad = -(\log_b c)$

Proportionality Rule

$\log_b a = k \log_c a$

$k = \log_b a / \log_c a = \log c / \log b$

eg $\log 5 / \log 2 = \ln 5 / \ln 2 \approx 2.32 \ldots$

Combination Rule

$\log_b a + k \quad = \log_b a + \log_b b^k$

and using product rule we absorb to

$= \log_b (a \times b^k)$

This is useful when solving exponential equations.

Exponential Rule

Recalling $\quad a = b^{\log a}$ we set $b = e$ so

$a = e^{\ln a}$

$a^x = e^{x \ln a}$

This rule also allows us to convert any exponential expression into a standard form. This is essential if we investigate a complex expression like $(a + ib)^{c + id}$

Complex Logs

The restriction that there are no logs of negative numbers can be breached if we move to complex logs written Log.

This is the briefest introduction

$z = r e^{i\theta} \Rightarrow z = e^{\ln r} e^{i\theta}$

$\text{Ln } z = \ln r + i\theta$ where $r = | z |$

but remembering we can add any integer multiple of $2\pi i$ to Ln z so the general solution is

$\text{Ln } z = \ln | z | + i (\theta + 2k\pi)$

It can be shown that the product rule still applies

$\text{Log} (w z) = \text{Log } w + \text{Log } z$

Footnote

Euler said "There are infinitely many logarithms of a positive number but only one of them is real".

Robert Goodhand

Logarithms

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